

# Mathematical analysis of antenna using FDTD (Finite difference time domain) method

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**Abstract:** - This paper gives the information about the technique called finite difference time domain method which is one of the powerful and easy computational methods to solve the electromagnetics problem using Maxwell's equation. The radiating structure simple rectangular micro strip patch antenna with resonant frequency 2.4 Ghz and dielectric constant of the substrate  $\epsilon_r$  3.38 is taken with the thickness of 1.524 mm , width of 60 mm , length of 60 mm. Here the conductor material for patch and ground plane is taken as PEC.

**Keywords:-** FDTD, Computational methods, Micro strip patch antenna

## I. INTRODUCTION

Before the digital computer was developed, the analysis and design of electromagnetic devices and structures were largely experimental. Once the computer and numerical languages such as FORTRAN came along, people immediately began using them to tackle electromagnetic problems that could not be solved analytically. This led to a flurry of development in a field now referred to as computational electromagnetics (CEM). Many powerful numerical analysis techniques have been developed in this area in the last 50 years. As the power of the computer continues to grow, so do the nature of the algorithms applied as well as the complexity and size of the problems that can be solved [2-3]. While the data gleaned from experimental measurements are invaluable, the entire process can be costly in terms of money and the manpower required to do the required machine work, assembly, and measurements at the range. One of the fundamental drives behind reliable computational electromagnetics algorithms is the ability to simulate the behavior of devices and systems before they are actually built.

## II. METHOD IMPLEMENTATION

The FDTD method is a direct solution to Maxwell's time dependent curl equation and it solves the problem in the time domain. It will be shown in following section that FDTD method involves the effect of the reflection and the radiation that are commonly avoided by the other method. Although the FDTD method was first proposed in 1966 by Yee [1] to solve problems in electromagnetics scattering, this kind of concept was initially unpopular but as the computational power became more accessible then the method started to gain

popularity in the area of micro wave and millimetre-wave research. With the continuous evolution in computer research and the reducing the cost for computational operation the FDTD method is being applied to problems associated with increasingly higher frequencies, from microwave to millimetre-wave to optics [4].

There are strong possibilities that FDTD method will become the most powerful, easy and versatile in solving problems involving electromagnetic wave interaction. The theory on the basis of the FDTD method is simple. To solve an electromagnetic problem, the idea is to simply discretize, both in time and space, the Maxwell's equations with central difference approximations. The originality of the idea of Yee resides in the allocation in space of the electric and magnetic field components, and the marching in time for the evolution of the procedure. To better understand the theory of the method, we will start considering a simple one-dimensional problem. Assume, at this stage, "free space" as propagation medium. In this case, Maxwell's equations can be written as

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\epsilon_0} \nabla \times \mathbf{H} \quad (1)$$

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu_0} \nabla \times \mathbf{E} \quad (2)$$

In the one-dimensional case, we can use only  $E_x$  and  $H_y$ , and (1), (2) can be rewritten

As

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\epsilon_0} \frac{\partial H_y}{\partial z} \quad (3)$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0} \frac{\partial E_x}{\partial z} \quad (4)$$

That represents a plane wave traveling in the z direction.

Yee's scheme consists in considering  $E_x$  and  $H_y$  shifted in space by half a cell and in time by half a time step when considering a central difference approximation of the derivatives. In such a case, equations (3) and (4) can be written as

$$\frac{E_x^{n+1/2}(k) - E_x^{n-1/2}(k)}{\Delta t} = -\frac{1}{\epsilon_0} \frac{H_y^n(k+1/2) - H_y^n(k-1/2)}{\Delta z} \quad (5)$$

$$\frac{H_y^{n+1}(k+1/2) - H_y^n(k+1/2)}{\Delta t} = -\frac{1}{\mu_0} \frac{E_x^{n+1/2}(k+1) - E_x^{n+1/2}(k)}{\Delta z} \quad (6)$$

Equations (5) and (6) show the usefulness of Yee’s scheme in order to have a central difference approximation for the derivatives. In particular, the left term in equation (5) says

that the derivative of the E field at time  $n\Delta t$  can be expressed as a central difference using E field values at times  $(n+1/2)\Delta t$  and  $(n-1/2)\Delta t$ . The right term in equations (5) approximates instead the derivative of the H field at point  $k\Delta x$  as a central difference using H field values at points  $(k+1/2)\Delta x$  and  $(k-1/2)\Delta x$ . This approximations oblige us to calculate always the E field values at points  $\dots, (k-1)\Delta x, k\Delta x, (k+1)\Delta x, \dots$  and times  $\dots, (n-3/2)\Delta t, (n-1/2)\Delta t, (n+1/2)\Delta t, \dots$  and to calculate always the H field values at points  $\dots, (k-3/2)\Delta x, (k-1/2)\Delta x, (k+1/2)\Delta x, \dots$  and times  $\dots, (n-1)\Delta t, n\Delta t, (n+1)\Delta t, \dots$

This scheme is known as “leap-frog” algorithm. Practically, it means that to approximate Maxwell’s equations in space and time using this algorithm, one should calculate first all H field values, then all E field values, remembering always that E and H are shifted also in space by half of the discretization  $\Delta x$ . Figure 1 shows schematically the algorithm.

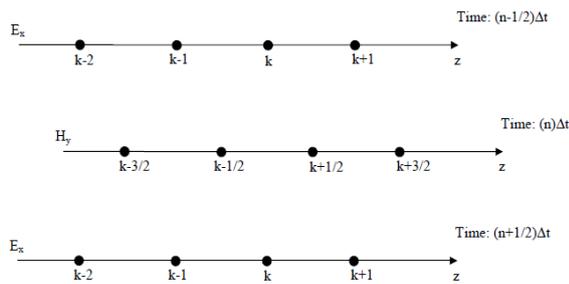


Figure 1. Yee’s one-dimensional scheme for updating EM fields in space and time.

$$E_x^{n+1/2}(k) = E_x^{n+1/2}(k) + \frac{\Delta t}{\epsilon_0 \Delta z} (H_y^n(k-1/2) - H_y^n(k+1/2)) \quad (7)$$

$$H_y^{n+1}(k+1/2) = H_y^n(k+1/2) + \frac{\Delta t}{\mu_0 \Delta z} (E_x^{n+1/2}(k) - E_x^{n+1/2}(k+1)) \quad (8)$$

The explicit FDTD equations can be derived from (5) and (6) obtaining

$$\tilde{E} = \sqrt{\frac{\epsilon_0}{\mu_0}} E$$

Equations (7) and (8), adopting this substitution and dropping the symbol “ $\sim$ ” from now on, become

$$E_x^{n+1/2}(k) = E_x^{n+1/2}(k) + \frac{1}{\sqrt{\mu_0 \epsilon_0}} \frac{\Delta t}{\Delta z} (H_y^n(k-1/2) - H_y^n(k+1/2)) \quad (10)$$

$$H_y^{n+1}(k+1/2) = H_y^n(k+1/2) + \frac{1}{\sqrt{\mu_0 \epsilon_0}} \frac{\Delta t}{\Delta z} (E_x^{n+1/2}(k) - E_x^{n+1/2}(k+1)) \quad (11)$$

These equations can be directly implemented in a computer code. Note that the “1/2” in equations (10) and (11) do not

need to be implemented in the computer code. The half a cell and half a time step are necessary in equations (10) and (11) just to remind us the physical definitions of E and H, and to remind us that E and H are actually offset by half a cell and half a time step. This information, anyway, will never appear in our coding. To implement the code for the calculation of the fields obeying to equations (10) and (11) we need to:

- a) Define the size KE of the arrays E and H that, once we have chosen the spatial resolution  $\Delta z$ , will correspond to the absolute size of the computational domain;
- b) Determine the time step necessary according to our resolution and excitation signal;
- c) Implement a cycle to compute the fields for a certain number of time steps. Within the cycle, we need to include:
  - d) A cycle to calculate the various  $E_x(K)$  according to equation (10) for all the cells of the domain KE;
  - e) The excitation signal at the source point KS;
  - f) A cycle to calculate the various  $H_y(K)$  according to equation (11) for all the cells of the domain KE; Note that in the computer code we do not need to include the information relative to the half a cell shift (i.e., the “1/2”) since this is only the interpretation that we need to give to the field, and does not correspond to any practical modification in the algorithm.

The problem is now: how to choose  $\Delta z$  and  $\Delta t$ . How fine should our resolution be? The first parameter to choose is generally the resolution  $Dz$ . It has been verified that at least 10 cells per wavelength are necessary to ensure an adequate representation. The wavelength to consider is the smallest wavelength in the simulation. We did not consider yet the presence in our simulations of dielectrics, but we will shortly see that the presence of dielectric will just slightly modify the FDTD scheme given by equations (10) and (11). Anyway, it is well known that the signal wavelength is smaller in dielectrics with higher dielectric constants and losses. Therefore, in the case we have to consider dielectrics, we should first calculate the wavelength in the dielectric with higher dielectric constant and losses, and then consider the cell size to be about ten times smaller than this value. Once the cell size has been chosen, the time step is also chosen according to stability considerations. For stability reasons, a field component cannot propagate more than one cell size in the time step  $\Delta t$ . This means that

$$\Delta t \leq \frac{\Delta z}{c_0} \quad (12)$$

Since, the wave travels at the speed of the light  $C_0$ . This is the stability condition for one dimensional problem. It can be proven that, in general, the stability condition (Courant condition) is given by

$$\Delta t \leq \frac{\Delta}{c_0 \sqrt{d}} \quad (13)$$

With  $d=1, 2$ , or  $3$  for one-, two-, or three-dimensional problems, respectively, and  $\Delta$  the smallest cell size. A

common choice for  $\Delta t$  (in one-, two-, or three-dimensional problems) is anyway given by

$$\Delta t = \frac{\Delta}{2 c_0} \quad (14)$$

### III. BOUNDARY CONDITIONS

From the previous discussion, it is not clear what happens at the mesh termination. Of course, we cannot simulate the propagation of the signal indefinitely, and we need to terminate somehow the FDTD grid [6]. The problem does not exist in the case of a spatially limited structure, like a waveguide, a resonator, etc., where we need to model a region that “trap” the field inside. In most of the problems, however, we need to simulate open space regions. In these cases, since our simulation region MUST be limited, we need to find a way to “simulate” the open space.

These boundary conditions are called RADIATION BOUNDARY CONDITIONS (RBCs) or ABSORBING BOUNDARY CONDITIONS (ABCs). In 1D (the formulation in eq. (10) and (11)) the problem can be easily solved. We can observe, in fact, that if we use  $\Delta t$  given by (14) ( $\Delta t = \Delta z / (2 c_0)$ ), since the field travels at the speed of the light  $c_0$ , in one time step the field will travel only half a cell. This means that to entirely cross one cell, two time steps are necessary. The absorbing boundary condition for the 1D case can be therefore expressed by

$$E_x^{n+1/2}(1) = E_x^{n-2+1/2}(2) \quad (15)$$

For the left side of the mesh and

$$E_x^{n+1/2}(KE) = E_x^{n-2+1/2}(KE-1) \quad (16)$$

For the right side of the mesh. With these conditions, in the 1D simulation described in the previous section the wave will be completely “absorbed” by the termination. Of course, “completely” means actually “relatively”, since for numerical errors some small reflections from the boundary (noise) will be observed. This is the simplest approach to derive an absorbing boundary condition. Several ABCs have been proposed in the literature that resort to analytical approximations of the wave equation or use fictitious absorbing materials to physically absorb the outgoing wave. One traditional ABC is that developed by G. Mur. The boundary condition proposed by Mur can be explained by considering the wave equation that the electromagnetic field obeys.

Considering the 1D case, we can write the following wave equation:

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{1}{c_0^2} \frac{\partial^2 E_x}{\partial t^2} = 0 \quad (17)$$

This equation can be rewritten as

$$\left( \frac{\partial}{\partial z} + \frac{1}{c_0} \frac{\partial}{\partial t} \right) \left( \frac{\partial}{\partial z} - \frac{1}{c_0} \frac{\partial}{\partial t} \right) E_x = 0 \quad (18)$$

Engquist and Madja have shown that an absorbing boundary condition for the left side of the grid [7] ( $z=0$ ) can be derived applying the condition

$$\left( \frac{\partial}{\partial z} - \frac{1}{c_0} \frac{\partial}{\partial t} \right) E_x = 0 \quad (19)$$

while on the right side of the mesh ( $z=KE$ ) the boundary condition can be derived by using

$$\left( \frac{\partial}{\partial z} + \frac{1}{c_0} \frac{\partial}{\partial t} \right) E_x = 0 \quad (20)$$

The idea of Mur is to express the derivatives around a point that is half a cell distant from the boundary calculated [8] at time  $n$ . In this case, we will be able to express the derivatives in equations (19) and (20) by using the electric field values at the points and times known by the simulation. Consider, for example, equation (19) that we should use to derive the boundary condition at the left side of the mesh ( $z=0$ ). We obtain, using central differences,

$$\frac{1}{\Delta z} (E_{x2}^n - E_{x1}^n) - \frac{1}{c_0 \Delta t} (E_{x1/2}^{n+1/2} - E_{x1/2}^{n-1/2}) = 0$$

and, therefore,

$$\frac{1}{\Delta z} \left( \frac{E_{x2}^{n-1/2} + E_{x2}^{n+1/2}}{2} - \frac{E_{x1}^{n-1/2} + E_{x1}^{n+1/2}}{2} \right) - \frac{1}{c_0 \Delta t} \left( \frac{E_{x1}^{n+1/2} + E_{x2}^{n+1/2}}{2} - \frac{E_{x1}^{n-1/2} + E_{x2}^{n-1/2}}{2} \right) = 0$$

Simplifying, and solving the equation for  $E_{x1}^{n+1/2}$ , we obtain

$$E_{x1}^{n+1/2} = E_{x2}^{n+1/2} + \frac{c_0 \Delta t - \Delta z}{c_0 \Delta t + \Delta z} (E_{x2}^{n+1/2} - E_{x1}^{n-1/2}) \quad (21)$$

Applying the same approximation to the right boundary, we obtain

$$E_{xKE}^{n+1/2} = E_{xKE-1}^{n+1/2} + \frac{c_0 \Delta t - \Delta z}{c_0 \Delta t + \Delta z} (E_{xKE-1}^{n+1/2} - E_{xKE}^{n-1/2}) \quad (22)$$

Equations (21) and (22) can be applied directly in the code to update the left and right points in the mesh, respectively.

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### IV. FDTD IN DIELECTRICS

So far, we have considered the one-dimensional FDTD algorithm in free space. How the presence of a dielectric will affect the algorithm? We can derive the modification we need to include in the algorithm by considering, again, Maxwell’s equations. For a generic medium, Maxwell’s equations can be written as

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\epsilon_0 \epsilon_r} \nabla \times \mathbf{H} - \frac{\sigma}{\epsilon_0 \epsilon_r} \mathbf{E} \quad (23)$$

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu_0} \nabla \times \mathbf{E} \quad (24)$$

In 1D, these equations can be rewritten as follows

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\epsilon_0 \epsilon_r} \frac{\partial H_y}{\partial z} - \frac{\sigma}{\epsilon_0 \epsilon_r} E_x \quad (25)$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0} \frac{\partial E_x}{\partial z} \quad (26)$$

Changing the variable  $E_x$  according to equation (9), we obtain

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\epsilon_r \sqrt{\mu_0 \epsilon_0}} \frac{\partial H_y}{\partial z} - \frac{\sigma}{\epsilon_0 \epsilon_r} E_x \quad (27)$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\sqrt{\mu_0 \epsilon_0}} \frac{\partial E_x}{\partial z} \quad (28)$$

At this point, we can just discretize equations (27) and (28) in the same we have done it for deriving equations (10) and (11). We obtain, after a few simple manipulations,

$$E_x^{n+1/2}(k) = \frac{1 - \frac{\sigma \Delta t}{2 \epsilon_0 \epsilon_r}}{1 + \frac{\sigma \Delta t}{2 \epsilon_0 \epsilon_r}} E_x^{n-1/2}(k) + \frac{1}{1 + \frac{\sigma \Delta t}{2 \epsilon_0 \epsilon_r}} \frac{\Delta t}{\sqrt{\mu_0 \epsilon_0} \epsilon_r \Delta z} (H_y^n(k-1/2) - H_y^n(k+1/2)) \quad (29)$$

$$H_y^{n+1}(k+1/2) = H_y^n(k+1/2) + \frac{1}{\sqrt{\mu_0 \epsilon_0} \Delta z} (E_x^{n+1/2}(k) - E_x^{n+1/2}(k+1)) \quad (30)$$

Equation (30) is identical to equation (11), since we did not consider the possibility to have magnetic materials. Magnetic materials can be included easily modify equation (28) first, and the discretizing it has done for deriving equation (28). Note that equations (29) and (30) are more general than equations (10) and (11). Equations (10) and (11) are obtained by simply imposing  $\epsilon_r=1$  and  $\sigma=0$  in equations (29) and (30) and, therefore, we can focus on these last two equations for developing the general 1D

### V. SIMULATION OF MICRO STRIP PATCH ANTENNA

As we saw that using YEE algorithm of simple 1D wave propagation with the additional information like boundary conditions like MUR[10][11] and the effect of the dielectric material present in the path of the wave propagation only in 1D. So, with help of the basic information we can also analysis the electromagnetic wave scattering in simple micro strip patch antenna [5] with the resonant frequency of 2.4 GHz and with the dielectric constant 3.38

The input impedance for this patch antenna can be calculated and can be simulated with the industrial and mathematical tool- MATLAB.

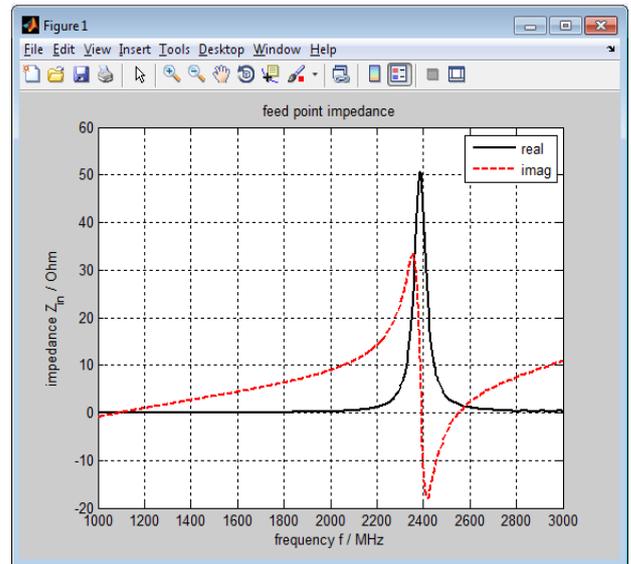


Fig 1: Measurement for the impedance  $z_{in}$  vs freq

Return loss is the most important part of while designing any antenna. So, the graph shows the return loss ( $S_{11}$ ) parameter Vs frequency. Most of the time simple FDTD method can take long time to get result so to improve the speed of the running code in Matlab must be concern of research. Sometime the complex but improved FDTD code [9] is used for the better result.

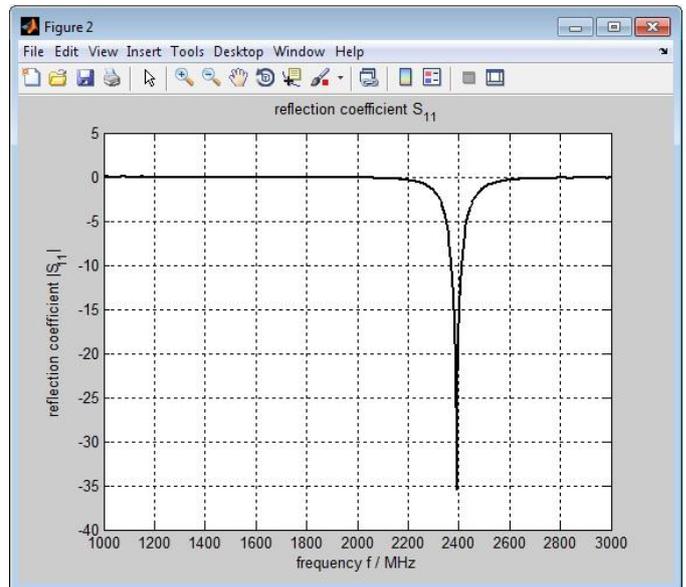


Fig. 2 Measurement for the impedance reflectionco. vs freq  
 Now by comparing these results with professional software like CST, HFSS, FEKO etc., we observe that, the same results are getting by simulating the antenna in MATLAB with the same dimension of the patch and the other important values like the dielectric constant and its height, width, and the length, and by calculating the appropriate values of patch regarding the resonant frequency, and the feeding technique.

We can also compare the radiation pattern of the MSA which is shown below,

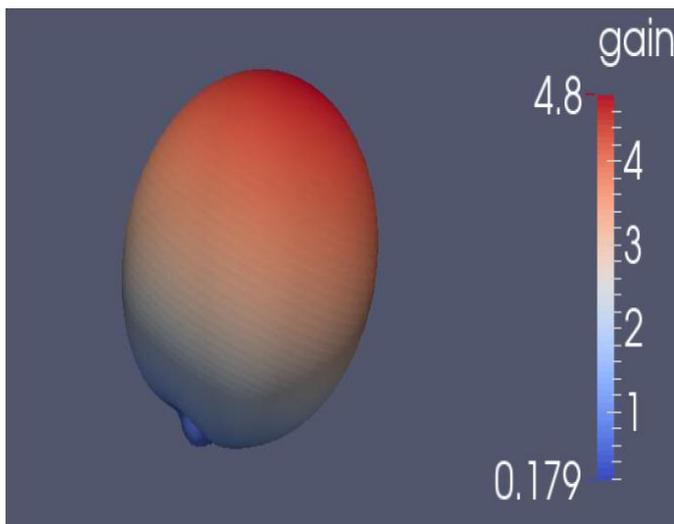


Fig.3 Gain

#### VI. CONCLUSION

Finally we can conclude that the any radiating structure like antenna or any physical phenomena which is relate to the electromagnetic scattering can be analysed by the powerful technique called FINITE DIFFERENCE TIME DOMAIN in MATLAB Software.

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