## Review of Three-Dimensional Finite Difference Time Domain Techniques

Deepak Kumar Soni<sup>1</sup> Bhagwat Kakde<sup>2</sup>

<sup>1</sup>M. E Scholar <sup>2</sup>Guide <sup>1,2</sup>Department of Electronics & Communication Engineering <sup>1,2</sup>RKDFIST, Bhopal

**Abstract**— A direct three-dimensional finite-difference timedomain (FDTD) method is applied to the full-wave analysis of various microstrip structures. The method is shown to be an efficient tool for modeling complicated microstrip circuit components and microstrip antennas. From the time-domain results the input impedance of a line-fed rectangular patch antenna and the frequency-dependent scattering parameters of a low-pass filter and a branch-line coupler are calculated. These circuits were fabricated and the measurements made on them are compared with the FDTD results and shown to be in good agreement. **Keywords**— FDTD,TSFS

#### I. INTRODUCTION

The finite-difference time-domain (FDTD) method is arguably the simplest, both conceptually and in terms of implementation, of the full-wave techniques used to solve problems in electromagnetic[1]. It can accurately tackle a wide range of problems. However, as with all numerical methods, it does have its share of artifacts and the accuracy is contingent upon the implementation. The FDTD method can solve complicated problems, but it is generally computationally expensive. Solutions may require a large amount of memory and computation time. The FDTD method loosely fits into the category of "resonance region" techniques, i.e., ones in which the characteristic dimensions of the domain of interest are somewhere on the order of a wavelength[5] in size. If an object is very small compared to a wavelength, quasi-static approximations generally provide more efficient solutions. Alternatively, if the wavelength is exceedingly small compared to the physical features of interest, ray-based methods or other techniques may provide a much more efficient way to solve the problem.[4]). The FDTD method employs finite differences as approximations to both the spatial and temporal derivatives that appear in Maxwell's equations (specifically Ampere's and Faraday's laws). Consider the Taylor series expansions of the function f(x) expanded about the point x0 with an offset of  $\pm \delta/2$ :

$$\begin{aligned} f\left(x_{0} + \frac{\delta}{2}\right) &= f(x_{0}) + \frac{\delta}{2}f'(x_{0}) + \frac{1}{2!}\left(\frac{\delta}{2}\right)^{2}f''(x_{0}) + \frac{1}{3!}\left(\frac{\delta}{2}\right)^{3}f'''(x_{0}) + \dots \\ f\left(x_{0} - \frac{\delta}{2}\right) &= f(x_{0}) - \frac{\delta}{2}f'(x_{0}) + \frac{1}{2!}\left(\frac{\delta}{2}\right)^{2}f''(x_{0}) - \frac{1}{3!}\left(\frac{\delta}{2}\right)^{3}f'''(x_{0}) + \dots \end{aligned}$$

where the primes indicate differentiation. Subtracting the second equation from the first yields

$$f\left(x_0 + \frac{\delta}{2}\right) - f\left(x_0 - \frac{\delta}{2}\right) = \delta f'(x_0) + \frac{2}{3!} \left(\frac{\delta}{2}\right)^{\circ} f'''(x_0) + \dots$$
  
Dividing by  $\delta$  produces

$$\frac{f(x_0 + \frac{\delta}{2}) - f(x_0 - \frac{\delta}{2})}{\delta} = f'(x_0) + \frac{1}{3!} \frac{\delta^2}{2^2} f'''(x_0) + \dots$$

Thus the term on the left is equal to the derivative of the function at the point x0 plus a term which depends on  $\delta 2$  plus an infinite number of other terms which are not shown. For the terms which are not shown, the next would depend on  $\delta 4$  and all subsequent terms would depend on even higher powers of  $\delta$ . Rearranging slightly, this relationship is often stated as

$$\left. \frac{df(x)}{dx} \right|_{x=x_0} = \frac{f\left(x_0 + \frac{\delta}{2}\right) - f\left(x_0 - \frac{\delta}{2}\right)}{\delta} + O(\delta^2)$$

The "big-Oh" term represents all the terms that are not explicitly shown and the value in parentheses, i.e.,  $\delta 2$ , indicates the lowest order of  $\delta$  in these hidden terms. If  $\delta$  is sufficiently small, a reasonable approximation to the erivative may be obtained by simply neglecting all the terms represented by the "big-Oh" term. Thus, the centraldifference approximation is given by

$$\frac{df(x)}{dx}\Big|_{x=x_0} \approx \frac{f\left(x_0 + \frac{\delta}{2}\right) - f\left(x_0 - \frac{\delta}{2}\right)}{\delta}.$$

Note that the central difference provides an approximation of the derivative of the function at x0, but the function is not actually sampled there. Instead, the function is sampled at the neighboring points  $x0+\delta/2$  and  $x0-\delta/2$ . Since the lowest power of  $\delta$  being ignored is second order, the central difference is said to have second-order accuracy or second-order behavior[5]. This implies that if  $\delta$  is reduced by a factor of 10, the error in the approximation should be reduced by a factor of 100 (at least approximately). In the limit as  $\delta$  goes to zero, the approximation becomes exact. One can construct higher-order central differences. In order to get higher-order behavior, more terms, i.e., more sample points, must be used. Appendix A presents the construction of a fourthorder central difference. The use of higher-order central differences in FDTD schemes is certainly possible, but there are some complications which arise because of the increased "stencil" of the difference operator. For example, when a PEC is present, it is possible that the difference operator will extend into the PEC prematurely or it may extend to the other side of a PEC sheet. Because of these types of issues, we will only consider the use of secondorder central difference.

### A. One-dimensional Simulation in Free Space

Electromagnetics is governed by the time-dependent Maxwell's curl equations, which in free space are

$$\frac{\partial \boldsymbol{E}}{\partial t} = \frac{1}{\varepsilon_0} \nabla \times \boldsymbol{H}$$

$$\frac{\partial \boldsymbol{H}}{\partial t} = -\frac{1}{\mu_0} \nabla \times \boldsymbol{E}$$
(1.1 a)
(1.1 b)

Eand H are vectors in three dimensions, but if we consider only one dimension

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon_0} \frac{\partial H_y}{\partial z}$$
(1.2 a)  
$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0} \frac{\partial E_x}{\partial z}$$
(1.2 b)

To put these equations in a computer, we approximate the derivatives with the "finite-difference" approximations:

$$\frac{\frac{E_x^{n+1/2}(k) - E_x^{n-1/2}(k)}{\Delta t}}{\frac{H_y^{n+1}(k+1/2) - H_y^{n}(k-1/2)}{\Delta t}}{= -\frac{1}{\mu_0} \frac{\frac{H_y^{n}(k+1/2) - H_y^{n}(k-1/2)}{\Delta x}}{\frac{H_y^{n+1/2}(k+1) - E_x^{n+1/2}(k)}{\Delta x}}$$
(1.3 a)

In these two equations, time is specified by the superscripts, i. e., "n" actually means a time  $t = \Delta t \cdot n$ , and "k" actually means the distance  $z = \Delta x \cdot k$ . (It might seem more sensible to use  $\Delta z$  as the incremental step, since in this case we are going in the z direction. However,  $\Delta x$  is so commonly used for a spatial increment that I will use  $\Delta x$ .) We rearrange the above equations to :

$$E_{x}^{n+1/2}(k) = E_{x}^{n-1/2}(k) - \frac{\Delta t}{\varepsilon_{0} \cdot \Delta x} \Big[ H_{y}^{n}(k+1/2) - H_{y}^{n}(k-1/2) \Big]$$

$$(1.4 a)$$

$$H_{y}^{n+1}(k+1/2) = H_{y}^{n}(k+1/2) - \frac{\Delta t}{\mu_{0} \cdot \Delta x} \Big[ E_{x}^{n+1/2}(k+1) - E_{x}^{n+1/2}(k) \Big]$$

$$(1.4 b)$$

Notice that the calculations are interleaved in both space and time. In Eq. (1.4 a), for example, the new value of  $E_x$  is calculated from the previous value of  $E_x$  and the most recent values of  $H_y$ . This is the fundamental paradigm of the finite-difference time-domain (FDTD) method Fig. 1.1) [1].

Eq. (1.4 a) and (1.4 b) look very similar. However,  $\mathcal{E}_0$  and  $\mu_0$  differ by several orders of magnitude:

$$\varepsilon_0 = 8.85 \times 10^8 \quad F/m,$$
$$\mu_0 = 4\pi \times 10^{-7} \quad H/m$$

Therefore,  $E_x$  and  $H_y$  will differ by several orders of magnitude. This is circumvented by making the following change of variables [2]:

$$\tilde{E} = \sqrt{\frac{\varepsilon_0}{\mu_0}} E$$

(1.5)

Substituting this into Eq. (1.4a) and (1.4b) gives

$$\tilde{E}_{x}^{n+1/2}(k) = \tilde{E}_{x}^{n-1/2}(k) - \frac{1}{\sqrt{\varepsilon_{0}\mu_{0}}} \frac{\Delta t}{\Delta x} \Big[ H_{y}^{n}(k+1/2) - H_{y}^{n}(k-1/2) \Big]$$
(1.6a)

$$H_{y}^{n+1}(k+1/2) = H_{y}^{n}(k+1/2) - \frac{1}{\sqrt{\varepsilon_{0}\mu_{0}}} \frac{\Delta t}{\Delta x} \left[ \tilde{E}_{x}^{n+1/2}(k+1) - \tilde{E}_{x}^{n+1/2}(k) \right]$$
(1.6b)

Now both E and H will have the same order of magnitude. We will call this "normalized" units. Physicist call this Gaussian units. Note that

$$\left[\sqrt{\frac{\varepsilon_0}{\mu_0}}\right] = \left[\frac{F/m}{H/m}\right]^{1/2} = \left[\frac{C/V}{Wb/A}\right]^{1/2}$$
$$= \left[\frac{C/V}{V/s-A}\right]^{1/2} = \left[\frac{A}{V}\right] = \left[\frac{1}{ohm}\right]$$

and

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} = \sqrt{1.42 \times 10^5} = 377 \,\Omega$$

This quantity is called the "impedance of free space." Once the cell size  $\Delta x$  is chosen, then the time step  $\Delta t$  is determined by

$$\Delta t = \frac{\Delta x}{2 \cdot c_0} \tag{1.7}$$

where  $\mathcal{C}_0$  is the speed of light in free space. Therefore,

$$\frac{1}{\sqrt{\varepsilon_0\mu_0}}\frac{\Delta t}{\Delta x} = c_0 \cdot \frac{\Delta x/2 \cdot c_0}{\Delta x} = \frac{1}{2}$$
(1.8)



Fig. 1.1: A diagram of the calculation of E and H fields in FDTD.

Re-writing Eq. (1.6 a) and (1.6 b) in C computer code gives the following:

- ex[k] = ex[k] + 0.5\*(hy[k-1] hy[k])(1.9 a) hy[k] = hy[k] + 0.5\*(ex[k] - ex[k+1])(1.9 b)
- Note that the n or n+1/2 or n-1/2 in the superscripts

is gone. Time is implicit in the FDTD method. In Eq. (1.9 a), the exon the right side of the equal sign is the previous value at n - 1/2, and the ex on the left side is the new value, n+1/2, which is being calculated. Position, however, is explicit. The only difference is that k + 1/2 and k - 1/2 are rounded off to k and k-1 in order to specify a position in an array in the program. Figure 1.2 illustrates a simulation in free space. The following things are worth noting:

- 1) The  $E_x$  and  $H_y$  values are calculated by separate loops, and they employ the interleaving described above.
- 2) After the  $E_x$  values are calculated, the source is calculated. This is done by simply specifying a value of

 $E_x$  at the point k = 1, and overriding what was previously calculated. This is referred to as a "hard source," because a specific value is imposed on the FDTD grid.

### B. Three-Dimensional Simulation

The original FDTD paradigm was described by the "Yee Cell," (Fig. 1.2), named, of course, after Kane Yee [1]. Note that the E and H fields are assumed interleaved around a cell whose origin is at the location I, J, K. Every E field is located 1/2 cell width from the origin in the direction of its orientation; every H field is offset 1/2 cell in each direction except that of its orientation.



Fig. 1.2: The Yee cell. Not surprisingly, we will start with Maxwell's equations

$$\frac{\partial \tilde{D}}{\partial t} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \cdot \nabla \times H$$

$$\tilde{D}(\omega) = \varepsilon_r^*(\omega) \cdot \tilde{E}(\omega) \quad (1.8)$$

$$\frac{\partial H}{\partial t} = -\frac{1}{\sqrt{\varepsilon_0 \mu_0}} \nabla \times \tilde{E}$$

$$(1.10)$$

Once again, we will drop the ~ notation, but it will always be assumed that we are referring to the normalized values.

Eqs. (1.8) and (1.10) produce six scalar equations, two of which are:

$$\frac{\partial D_z}{\partial t} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right)$$
(1.11 a)  
$$\frac{\partial H_z}{\partial t} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \left( \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right)$$
(1.11b)

The first step is to take the finite difference approximations.

# II. COMPUTER IMPLEMENTATION OF A ONE-DIMENSIONAL FDTD SIMULATION

The assignment operator " In C, the following is a erfectly valid statement

$$a = a+b;$$

In the usual mathematical sense, this statement is only true if b were zero. However, to a computer this statement means take the value of b, add it to the old value of a, and place the result back in the variable a. Essentially we are updating the value of a. In C this statement can be written more tersely as

a += b;

When writing a computer program to implement the FDTD algorithm, one does not bother trying to construct a program that explicitly uses offsets of one-half. Nodes are stored in arrays and, as is standard practice, individual array elements are specified with integer indices. Thus, the computer program (or, perhaps more correctly, the author of the computer program) implicitly incorporates the fact that electric and magnetic fields are offset while using only integer indices to specify location. As you will see, spatial location and the array index will be virtually synonymous. For example, assume two arrays, ez and hy, are declared which will contain the Ez and Hy fields at 200 nodes

double ez[200], hy[200], imp0=377.0;

Fig. 2.1: A one-dimensional FDTD space showing the assumed spatial arrangement of the electric- and magnetic-field nodes in the arrays ez and hy. Note that an electric-field node is assumed to exist to the left of the magnetic-

field node with the same index.

The variable imp0 is the characteristic impedance of free space and will be used in the following discussion (it is initialized to a value of 377.0 in this declaration). One should think of the elements in the ez and hy arrays as being offset from each other by a half spatial step even though the array values will be accessed using an integer index. It is arbitrary whether one initially wishes to think of an ez array element as existing to the right or the left of an hy element with the same index (we assume "left" corresponds to descreasing values of x while "right" corresponds to increasing values). Here we will assume ez nodes are to the left of hy nodes with the same index. This is illustrated in Fig. 2.1 where ez[0] is to the left of hy[0], ez[1] is to the left of hy[1], and so on. In general, when a Courier font is used, e.g., hy[m], we are considering an array and any offsets of one-half associated with that array are implicitly understood. When Times-Italic font is use, e.g.,  $H_y^{q+\frac{1}{2}} [m+\frac{1}{2}]_{\text{We}}$ 

are discussing the field itself and offsets will be given explicitly.

Assuming a Courant number of unity (Sc = 1), the node hy[1] could be updated with a statement such as hy[1] = hy[1] + (ez[2] - ez[1]) / imp0; In general, any magnetic-field node can be updated with hy[m] = hy[m] + (ez[m + 1] - ez[m]) / imp0; For the electric-field nodes, the update equation can be written ez[m] = ez[m] + (hy[m] - hy[m - 1]) \* imp0;

These two update equations, placed in appropriate loops, are the engines that drive an FDTD simulation. However, there are a few obvious pieces missing from the puzzle before a useful simulation can be performed. These missing pieces include

 Nodes at the end of the physical space do not have neighboring nodes to one side. For example, there is no hy[-1] node for the ez[0] node to use in its update equation. Similarly, if the arrays are declared with 200 element, there is no z[200] available for hy[199] to use in its update equation (recall that the index of the last element in a C array is one less than the total number of elements—the array index represents the offset from the first element of the array). Therefore a standard update equation cannot be used at these nodes.

- 2) Only constant impedance is used so only a homogeneous medium can be modeled (in this case free space).
- 3) As of yet there is no energy present in the field. If the fields are initially zero, they will remain zero forever.

The first issue can be addressed using absorbing boundary conditions (ABC's). There are numerous implementations one can use. In later material we will be consider only a few of the more popular techniques. The second restriction can be removed by allowing the permittivity and permeability to change from node to node. However, in the interest of simplicity, we will continue to use a constant impedance for a little while longer. The third problem can be overcome by initializing the fields to a nonzero state. However, this is cumbersome and typically not a good approach. Better solutions are to introduce energy via either a hardwired source, an additive source, or a totalfield/scattered-field (TFSF) boundary. We will consider mplementation of each of these approaches.

### III. HISTORY

Finite difference schemes for time-dependent PDEs have been employed for many years in computational fluid dynamic sproblems,<sup>[1]</sup> including the idea of using centered finite difference operators on staggered grids in space and time to achieve second-order accuracy.<sup>[1]</sup> The novelty of Kane Yee's FDTD scheme, presented in his seminal 1966 paper,<sup>[2]</sup>was to apply centered finite difference operators on staggered grids in space and time for each electric and magnetic vector field component in Maxwell's curl equations. The descriptor "Finite-difference time-domain" and its corresponding "FDTD" acronym were originated by Allen Taflovein 1980. Since about 1990, FDTD techniques have emerged as primary means to computationally model many scientific and engineering problems dealing with electromagnetic ave interactions with material structures. Current FDTD modeling from near-DC (ultralowapplications range frequency geophysics involving the entire Earth-ionosphere through microwaves (radar waveguide) signature technology.

### IV. CONCLUSION

The basis and applications of the finite-difference time domain (FD-TD) numerical modeling approach for Maxwell's equations. FD-TD is very simple in concept and execution. However, it is remarkably robust, providing highly accurate modeling predictions for a wide variety of electromagnetic wave interaction problems. The accuracy and breadth of FD-TD applications will be illustrated by a number of two- and three-dimensional examples. The objects modeled range in nature from simple geometric shapes to extremely complex aerospace and biological systems. In all cases where rigorous analytical, code-tocode, or experimental validations are possible, FD-TD predictive data for penetrating and scattered near fields as well as radar cross sections are in excellent agreement with the benchmarks. It will also be shown that opportunities are arising in applying FD-TD to model rapidly time-varying systems, microwave circuits, and inverse scattering. With continuing advances in FD-TD modeling theory as well as continuing advances in supercomputer technology, there is a strong possibility that FD-TD numerical modeling will occupy an important place in high-frequency engineering electromagnetics as we move into the 1990s.

### REFERENCES

- [1] . J. von Neumann and RD Richtmyer (March 1950). "A method for the numerical calculation of hydrodynamic shocks". Journal of Applied Physics 21: 232–237.
- [2] Kane Yee (1966). "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media". IEEE Transactions on Antennas and Propagation 14 (3): 302–307..
- [3] A. Taflove (1980). (PDF). IEEE Transactions on Electromagnetic Compatibility 22 (3): 191–202.
- [4] Susan C. Hagness (2005)d Artech House Publishers.
- [5] Adapted with permission from Taflove and Hagness (2005).
- [6] Richard Courant, Kurt Otto Friedrichs, and Hans Lewy (1928). (in German)100 (1): 32–74..
- [7] G. G. O'Brien, M. A Hyman, and S. Kaplan (1950). "A study of the numerical solution of partial differential equations". Journal of Mathematical Physics 29 (1): 223–251.
- [8] Dong-Hoa Lam (1969). "Finite Difference Methods for Electromagnetic Scattering Problems". Missispi State University, Interaction Notes 44.
- [9] A. Taflove and M. E. Brodwin (1975). "(PDF). IEEE Transactions on Microwave Theory and Techniques23 (8):
- [10] A.Taflove and M. E. Brodwin (1975). (PDF). IEEE Transactions on Microwave Theory and Techniques23 (11): 888–896.
- [11] R. Holland (1977). "Threde: A free-field EMP coupling and scattering code". IEEE Transactions on Nuclear Science 24 (6): 2416–2421..
- [12] K. S. Kunz and K. M. Lee (1978). "A threedimensional finite-difference solution of the external response of an aircraft to a complex transient EM environment". IEEE Transactions on Electromagnetic Compatibility 20 (2): 333–341..
- [13] G. Mur (1981).. IEEE Transactions on Electromagnetic Compatibility 23(4): 377–382.
- [14] K. R. Umashankar and A. Taflove (1982). (PDF). IEEE Transactions on Electromagnetic Compatibility 24 (4): 397–405.
- [15] A. Taflove and K. R. Umashankar (1983). "(PDF). IEEE Transactions on Electromagnetic Compatibility 25 (4): 433–440.
- [16] Z. P. Liao, H. L. Wong, B. P. Yang, and Y. F. Yuan (1984). "A transmitting boundary for transient wave analysis". Scientia Sinica a27: 1063–1076.
- [17] W. Gwarek (1985). IEEE Transactions on Microwave Theory and Techniques 33 (10): 1067–1072.

- [18] D. H. Choi and W. J. Hoefer (1986). IEEE Transactions on Microwave Theory and Techniques 34 (12): 1464–1470..
- [19] G. A. Kriegsmann, A. Taflove, and K. R. Umashankar(1987). (PDF). IEEE ransactions on Antennas and Propagation 35 (2): 153–161..
- [20] T. G. Moore, J. G. Blaschak, A. Taflove, and G. A. Kriegsmann (1988). (PDF). IEEE Transactions on Antennas and Propagation 36 (12): 1797–1812.