

Optimum Design of Finite Impulse Response Filters Using PM Algorithm

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Abstract— Finite impulse response filters play an important role in field of signal and system. The paper deals with the optimum design of finite impulse response low pass and high pass filters using PM algorithm. The analysis and synthesis optimum filter design ensures perfect transmission and reception of digital signals. Here we have designed a low pass filter . We also designed a high pass filter at the analysis and synthesis stage using complex algorithms called as PM algorithm. The result shows.....

Keywords— FIR analysis filter, FIR synthesis filter, low pass filter, high pass filter, PM algorithm, Pass band, Stop band, Transition band .

I. INTRODUCTION

Digital signal processing is an area of science and engineering that has developed rapidly over the past 40 years. Also there is often a higher order of precision achievable with digital hardware and software compared with analog circuits and signal processing system for all these reason there has been an explosive growth in digital signal processing theory and application over the past decades. A filter is used to reduce the noise and interfaces corrupting signal with a desired information bearing signal. In this case the filter performs some operations on the signal, which has the effect of reducing (filtering) the noise and interference from the desired information bearing signal.[1] In addition ,digital filters are widely used in the fields of automatic control telecommunication speech processing and many more. Basically filters refers to a frequency selective devices which extracts the useful portion of input signal lying within its operating frequency range and could be contaminated with random noise due to unavoidable circumstances. Analog filters are implemented with discrete time signal to reduce or enhance the desired feature of the applied signal. Digital filters are superior to their analog counterpart due to its wide range of applications and better performance. The advantages of digital filters over analog filters are small physical size, high accuracy and reliability.[3]

Digital filters is one of the most powerful tools of digital signal processing . Digital filters are capable of performance energy compaction specification such as ability to achieve multi-rate operations and extract linear phase that would at best be extremely difficult, if not impossible to achieve with an analog implementation . Digital filters are of two types finite impulse response (FIR) and infinite impulse response (IIR).

Traditional methods for such filters design is filters coefficients selection involve windowing functions as Butterworth, Chebyshev , Kaiser for FIR filters. and which can further be transformed to digital low pass, high pass , band pass or band stop filters using transformation technique, such as bilinear transformation for IIR filters ,

Remez exchange algorithm proposed by Parks McClellan (called PM algorithm)is better than all other traditional methods for design of FIR filters several methods has too limitations of computational complexity and high passband ripples.[2]

Such filters have great error and need optimization i.e. minimization of mean square error between desired response and estimated filter output. Steepest descent method of optimization can approximation any kind of frequency response of linear phase FIR filters but the transition width is to be compromised which is not acceptable. The other classical gradient based optimization is not new and several efforts have already been made for optimum filter design.[3]

II. DESIGN OF FIR LOW PASS FILTERS

First It is necessary to specify passband (s) stop band (s) and transition band (s) when designing a frequency selective filters. In passband (s) frequency are needed, to be passband unattenuated. In stop band (s) frequencies are completely attenuated, Transition band frequencies which are lying between the passband(s) and stopband (s). In practical , the magnitude is not necessary to be constant in the passband of a filter .[6] A small amount of ripple is usually allowed in the passband . Similarly, the filter response need not to be zero in the stopband. A small non zero value is also tolerable in the stopband. The frequency ω_p denotes the edge of passband and the stop band edge frequency ω_s and ω_t is the width of the transition band $\omega_t = \omega_s - \omega_p$. [6] The ripple in the passband of the filter is denoted as δ_p and the magnitude of the filters varies from $1 - \delta_p$ to $1 + \delta_p$, δ_s is the ripple in the stopband, usually we use a logarithms scale to show the frequency response, hence the ripple in the passband is $20 \log \delta_p$ dB and the ripple in the stopband is $20 \log \delta_s$ dB.[6]

Design of Linear -Phase FIR Filter using Windows was the first method. Window functions such as Rectangular, Bartlett Hanning, Hamming, Blackman Kaiser Window were experimented. The frequency sampling method and Optimum Equiripple methods were also developed in the 1970s. Various algorithm are used for optimization.

The frequency sampling method is better then the window method in the real valued Frequency Response characteristics $H_r(\omega)$ is specifically which can be either zero or unity at all frequency except the transition band.

The chebyshev approximation method offers complete control of the filters requirement as a result , this method is more preferable than the other two it is based on the Remez Exchange algorithm which minimize the error with respect to the max-norm.

To determine the coefficients of linear phase Fir filters of length $M=16$ which has a symmetric unit sample response and a frequency response that satisfy the condition.

The filter specifications which are used to design a low pass filter are filter types, sample rate, passband gain, stopband gain, low band edge, high band edge etc. Parameters of low pass filters are as follows:

Filter types= low pass filter
Sample rate=66000Hz
Passband Gain=1dB
Stopband Gain= -60dB
Low band Edge=6010Hz
High band Edge=15050Hz

Since h(n) is symmetric and the frequency are selected to correspond to the case $\alpha=0$, we use the corresponding formula to evaluate h(n)

$$H_r\left(\frac{2\pi k}{15}\right) = \begin{cases} 1 & k = 0,1,2,3 \\ 0.4 & k = 4 \\ 0 & k = 5,6,7 \end{cases}$$

$$H(k) = G(k)e^{j\frac{\pi k}{M}} \quad k = 0,1, \dots, M-1$$

$$G(k) = (-1)^k H_r\left(\frac{2\pi k}{M}\right), \quad G(k) = -G(M-k)$$

$$h(n) = \frac{1}{M} \left\{ G(0) + 2 \sum_{k=1}^U G(k) \cos \frac{2\pi k}{M} \left(n + \frac{1}{2}\right) \right\}$$

$$U = \begin{cases} \frac{M-1}{2} & M \text{ odd} \\ \frac{M}{2} - 1 & M \text{ even} \end{cases}$$

$$G(k) = (-1)^k H_r\left(\frac{2\pi k}{15}\right), \quad k = 0,1, \dots, 8$$

Coefficients values of h(n)		PM algorithm
h(0)	h(15)	-0.004907035546594173.....
h(2)	h(14)	-0.017465456634511795.....
h(3)	h(13)	-0.03233553432076823.....
h(4)	h(12)	-0.030641825988093625.....
h(5)	h(11)	0.0102362728155300816.....
h(6)	h(10)	0.09557752181436623.....
h(7)	h(9)	0.19826668118823884.....
h(8)	-----	0.26913196740330014.....

Table1 shows the value of h(n) in PM algorithm.

III. DESIGN OF FIR HIGH PASS FILTERS

With the help of low pass filter and having their parameters values in the receiving end to complete the signal we design high pass filter.

To determine the coefficients of linear phase FIR filter of length M=16 which has a symmetric unit samples response and a frequency response that satisfies the conditions .Since we have a filters specifications which are used to design a low pass filter, filter types, sample rate, passband gain, stopband gain, low band edge, High band Edge .

Parameters of high pass filters are as follows:

Filter types= High pass filter

Sample rate=66000Hz

Passband Gain=1dB

Stopband Gain= -60dB

Low band Edge=6010Hz

High band Edge=15050Hz

Since h(n) is symmetric and the frequency are selected to correspond to the case $\alpha=0$, we use the corresponding formula to evaluate h(n)

Coefficient values for h(n)		PM algorithm
h(1)	h(14)	0.00694798115342717
h(2)	h(13)	-0.03955489373828058
h(3)	h(12)	0.02111111109125235
h(4)	h(11)	0.06029422885979434
h(5)	h(10)	0.02909310982271561
h(6)	h(9)	-0.10926842167740647
h(7)	h(8)	-0.28560970160279434.

IV. METHODOLOGY FOR ALGORITHM

To understand the PM algorithm. First note that

$$E(\omega_i) = C \cdot (-1)^i \left| |E(\omega)| \right|_{\infty} \quad i = 1 \dots L$$

.....equation(1)

Can be written as follows ,let δ represents $C \left| |E(\omega)| \right|_{\infty}$ then equation (1) becomes.

$$W(\omega_i)[H_{dr}(\omega_i) - P(\omega_i)] = (-1)^i \delta \quad i = 0,1, \dots, L+1$$

.....equation(2)

$$P(\omega_i) + \frac{(-1)^i \delta}{W(\omega_i)} = H_{dr}(\omega_i) \quad i = 0,1, \dots, L+1$$

.....equation(3)

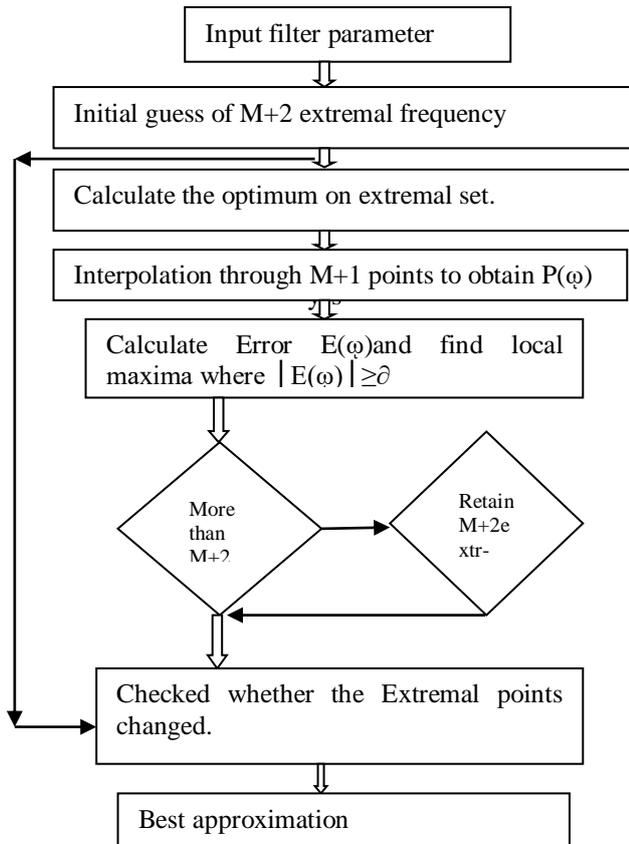
Or

$$\sum_{k=0}^L a(k) \cos(k\omega_i) + \frac{(-1)^i \delta}{W(\omega_i)} = H_{dr}(\omega_i) \quad i = 0,1, \dots, L+1$$

.....equation(4)

If set of extremal points in the alternation theorem were known in advance . Then the solution can be found by solving the system of equation (4). The system (4)

represents an interpolation problem which in matrix form become[3]



This is not a typical interpolation problem there are M+1 coefficient a(n). but there are L=M+2 interpolation points. However the values δ is taken as a variable in this interpolation problem, so that there are in fact total L variable and L equations.

The problem of obtaining the filters that minimize the Chebyshev criteria then becomes one of the finding the correct set of points over which to solved the interpolation problem (equation 4). In the process one obtain δ .

The PM algorithms proceeds by iteratively

- (1) Solving the interpolation problem in the process one obtain δ .

Solving the interpolation problem over a specified set of L points.

- (2) Updating the references set (by an exchange procedure) the initial reference set can be taken by L points uniformly spaced in $(0, \Pi)$ (excluding region where $W(\omega)$ is zero) convergence is achieved when

$$\frac{||E_k(\omega)||_{\infty} - |\delta_k|}{||E_k(\omega)||_{\infty}} < \epsilon \dots \dots \dots \text{equation(5)}$$

Where ϵ is a small number (like 10^{-6}) indicates the numerical accuracy desired [4].

In practical the algorithms is implemented by discretizing the frequency variable ω . Typically a uniform array of points.

$$\omega_k = \Pi/L \cdot k \quad 0 \leq k \leq L \dots \text{equation(6)}$$

V. ALGORITHMS

Steps of PM Algorithms:

- (1) The initialization steps.
- (2) The interpolation steps.
- (3) Updating the reference set.

In initialization step, the algorithms can be initialized by selecting any L frequency points between 0 and Π for which the weighting functions $W(\omega)$ is not zero.[7]

For example, one can choose the initial frequencies to be uniformly spaced over the region where $W(\omega) > 0$.

In the interpolation step, it requires solving the linear system. It can be treated as a general linear system however, there do exist fast algorithm for solving the system. Equation 7

In Updating the reference set, after interpolation step is performed the weighted error function is computed and a new reference set $\omega_1, \dots, \omega_L$ is found such that it satisfy the following updated criteria[3].

- (1) The current weight error function $E(\omega)$ alternates sign on the new reference set. $|E(\omega_i)| \geq |\delta|$ for each points ω_i of the new reference set.
- (2) $|E(\omega_i)| > |\delta|$ for at least one point ω_i of the new reference set.

PARAMETER USED IN PM ALGORITHM

A. To solve PM algorithm parameter

Alternation theorem-

Let S be a compact subset of interval $[0, \Pi]$. A necessary and sufficient condition for

$$P(\omega) = \sum_{k=0}^L a(k) \cos \omega k$$

to be the unique, best weighted chebyshev approximation to $H_{dr}(\omega)$ in S, is that the error function $E(\omega)$ exhibit at least L+2 extremal frequencies in S. That is, there must exist at least L+2 frequencies $\{\omega_i\}$ in S such that $\omega_1 < \omega_2 < \dots < \omega_{L+2}$,

$$E(\omega_i) = -E(\omega_{i+1})$$

and

$$|E(\omega_i)| = \max_{\omega \in S} |E(\omega)| \quad i = 1, 2, \dots, L + 2$$

We note that the error function $E(\omega)$ alternates in sign between two successive extremal frequencies. Hence the theorem is called alternation theorem.

The alternation theorem guarantees a unique solution for the Chebyshev optimization problem. At the desired extremal frequencies $\{\omega_n\}$, we have the set of equations

$$W(\omega_i)[H_{dr}(\omega_i) - P(\omega_i)] = (-1)^i \delta \quad n = 0, 1, \dots, L + 1$$

Where δ represents the maximum value of the error function $E(\omega)$. In fact, if we select $W(\omega)$ as indicated, it follows that $\delta = \delta_2$.

The set of linear equations can be arranged as

$$P(\omega_i) + \frac{(-1)^i \delta}{W(\omega_i)} = H_{dr}(\omega_i) \quad i = 0, 1, \dots, L + 1$$

or equivalently in the form

$$\sum_{k=0}^L a(k) \cos(k\omega_i) + \frac{(-1)^i \delta}{W(\omega_i)} = H_{dr}(\omega_i) \quad i = 0, 1, \dots, L + 1$$

if we treat $[a(k)]$ and δ as the parameter to be determined then above equation can be expressed in matrix form as

$$\begin{bmatrix} 1 & \cos\omega_1 & \dots & \cos M\omega_1 & 1/W(\omega_1) \\ 1 & \cos\omega_1 & \dots & \cos M\omega_1 & -1/W(\omega_2) \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \cos\omega_1 & \dots & \cos M\omega_1 & -(-1)^R/W(\omega_R) \end{bmatrix} \begin{bmatrix} a(0) \\ a(1) \\ \dots \\ a(m)\delta \end{bmatrix} = \begin{bmatrix} H_{dr}(\omega_1) \\ H_{dr}(\omega_2) \\ \dots \\ H_{dr}(\omega_R) \end{bmatrix}$$

Define to solve for the parameter we use an iterative algorithm called the PM algorithm in which [3] we begin by guessing at the set of extremal frequencies, determine $P(\omega)$ and δ and then compute the error function $E(\omega)$ we determine another set of $L+2$ extremal frequencies and repeat the process iteratively until it converges to the optimal set of extremal frequencies. Although the matrix inversion is time consuming and inefficient [7]

A more efficient procedure, suggested in the paper by Rabiner et al (1975), is to compute δ analytically, according to the formula

$$\delta = \frac{\gamma_0 \hat{H}_{dr}(\omega_0) + \gamma_1 \hat{H}_{dr}(\omega_1) + \dots + \gamma_{L+1} \hat{H}_{dr}(\omega_{L+1})}{\frac{\gamma_0}{W(\omega_0)} - \frac{\gamma_1}{W(\omega_1)} + \dots + \frac{(-1)^{L+1} \gamma_{L+1}}{W(\omega_{L+1})}} \quad \dots \dots \dots \text{equation(9)}$$

The expression for δ in equation (9) follows immediately from the matrix equation. Thus with an initial guess at the $L+2$ extremal frequencies, we compute δ .

Now since $P(\omega)$ is a trigonometric polynomial of the form

$$P(\omega) = \sum_{k=0}^L a(k)x^k \quad , x = \cos\omega$$

And since we know that [3] the polynomial at the points

$$X_n = \cos\omega_n \quad , \quad n=0, 1, \dots, L+1$$

Has the corresponding values

$$P(\omega_n) = \hat{H}_{dr}(\omega_n) - \frac{(-1)^n \delta}{W(\omega_n)} \quad , n = 0, 1, \dots, L + 1$$

.....equation (10)

We can use the Lagrange interpolation formula for $P(\omega)$. Thus $P(\omega)$ can be expressed as

$$P(\omega) = \frac{\sum_{k=0}^L P(\omega_k) \left[\frac{\beta_k}{x-x_k} \right]}{\sum_{k=0}^L \left[\frac{\beta_k}{x-x_k} \right]}$$

.....equation (11)

Where $P(\omega_n)$ is given by (Equation 10), $x = \cos\omega$, and

$$X_k = \cos\omega_k, \quad \dots \dots \dots \text{equation(12)}$$

Having the solution for $P(\omega)$, we can now compute the error function $E(\omega)$ from

$$E(\omega) = \hat{W}(\omega)[\hat{H}_{dr}(\omega) - P(\omega)] \quad \dots \dots \dots \text{equation (13)}$$

On a dense set of frequency points usually a number of points equal to $16M$, where M is the length of the filter, suffices. If $|E(\omega)| \geq \delta$ for some frequencies on the dense set, then a new set of frequencies corresponding to the $L+2$ largest peaks of $|E(\omega)|$ are selected and the computational procedure beginning with (equation 11) is repeated. [3] Since the new set of $L+2$ extremal frequencies is selected to correspond to the peaks of the error function $|E(\omega)|$. [4] The algorithm forces δ to increase in each iteration until it converges to the upper bound and hence to the optimum solution for the Chebyshev approximation problem. In other words, when $|E(\omega)| \leq \delta$ For the frequencies on the dense set the optimal solution has been found in terms of the polynomial $H(\omega)$. A flow chart of the algorithm is shown above and is due to PM.

Once the optimal solution has been obtained in terms of $P(\omega)$, the unit samples response $h(n)$ can be computed directly, without having to compute the parameters $(a(k))$. [1] In effort, we have determined

Which can be evaluated at $\omega = 2\pi k/M$, $k=0, 1, \dots, (M-2)/2$ for M odd, or $M/2$ for M even

A computer program written by Parks and McClellan (1972b) is available for designed linear phase FIR filters based on the Chebyshev approximation criterion and implemented with the PM algorithm. [8] This program can be used to design low pass and band pass filters, differentiators.

VI. RESULTS

The PM algorithm is simulated using MATLAB for the designed Low pass finite impulse response filter for

order(n) taken to be 15 The actual filters designed used all the parameter of ideal filter here as

$$H_d = \begin{cases} 1 & \text{for } 0 \leq \omega \leq 0.01335 \\ 0 & \text{for } 0.01335 \leq \omega \leq 0.02667 \end{cases}$$

Where $\omega_c = 0.13335$ represents cut off frequencies of the filters in the Hertz from the passband and stopband Edge frequencies of the filters of the filters are calculated as And having transfer function

$$H(z) = Y(z)/X(z)$$

$$= b(1)+b(2)Z^{-1}+\dots\dots+b(15)Z^{-12}$$

$$\omega_p = \omega_c - 0.01 = -0.01$$

$$\omega_s = \omega_c + 0.01 = 0.02667$$

The same are used in PM exchange algorithm as mentioned in design of low pass filters.

The algorithm is run 100 times and the best and average results are recorded the best optimized filters coefficient obtained for designed the desired LP filters , the best coefficients values among all the 100 runs.

The various performance parameters of the designed filters recorded to show its efficiency the designed linear phase LP FIR filter provide maximum stopband attenuation as high as which is greater than that achieved by other results reported in designed.

So, using PM algorithms we achieved as our desired results Table 1 shows parameter of low pass and High pass filter. In PM exchange algorithm.

Filter types	Low pass filter	High pass filter
Sample rate	66000Hz	66000Hz
Passband Gain	1dB	1dB
Stop band Gain	-60dB	-60dB
Low band Edge	6010Hz	6010Hz
High band Edge	15050Hz	15050Hz

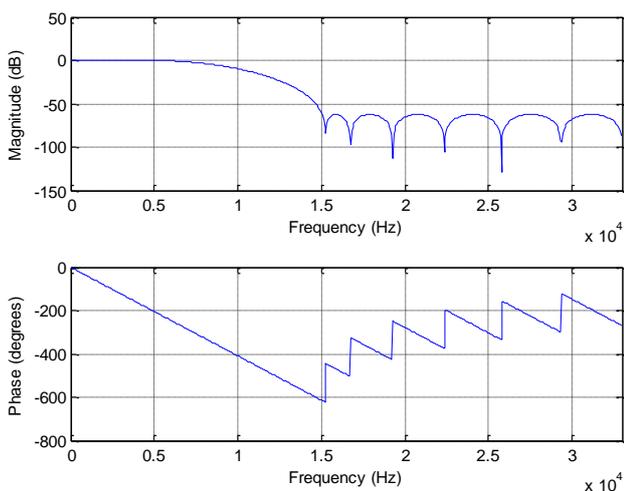
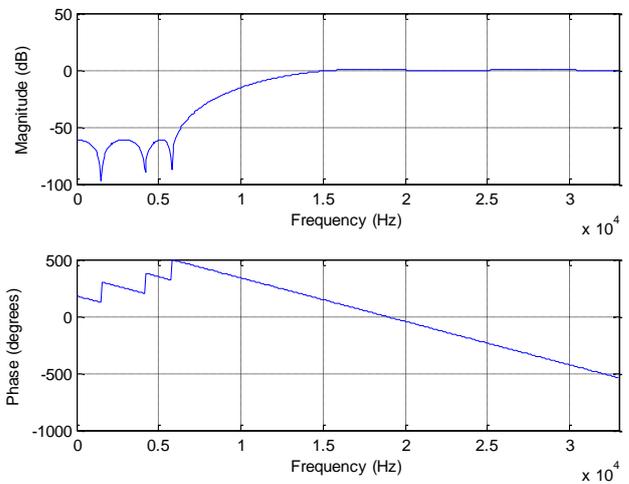


Figure (2) shows frequency magnitude graph of low pass filter

Co-efficient of filter	LPF	HPF

Filter order	15	14
Pass band Edge	0.01	0.01

Table(2) shows filter design results of LPF nad HPF



Figure(2) shows frequency magnitude graph of high pass filter

VII. CONCLUSION

Filter have advantages in different domain so, in this work the design of low pass and high pass filter using PM algorithm completed.

Therefore FIR digital filters should be choosing to use when calculations accuracy is the much requirement. In another way on the base of considering the calculations speed and calculations accuracy the researchers argue that using FIR low pass filter with order 15 is the optimal solution became FIR digital filter order 15 has capable both accuracy and speed.

Transfer function of filters using PM algorithms are accurate as possible in this method.

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