FIR QMF Filter Design

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Abstract— *various research techniques are worked on for designing a wavelet. Sub-band coding method consists of a sequence of filters and sub-sampling processes this paper deal with the characteristics of analysis low pass and high pass filters along with synthesis low pass and high pass filters. The design of FIR low pass filter enables the design of analysis high pass and synthesis low pass and synthesis high pass filters. Low pass and high pass linear phase filters are designed which satisfied the conditions of QMF.*

Keywords— Wavelet, Filter, Analysis Filter, Synthesis Filter, DWT, Image Restoration

I. Introduction

The image analysis especially image compression has been greatly impacted by wavelet transform. Our interests center on study of wavelet transform for exploitation in the field of image compression. Wavelet transform has high energy compaction property which makes it suitable for image compression[1]. The energy compaction property of the transform decides the amount of compression that can be achieved. The progressive reconstruction property of wavelet transform makes it a power tool for image and video compression.

II. Dwt

Wavelet series is called 'DWT', if the signal, scaling functions and 'wavelet's are discrete in time. The DWT of a sequence consists of two series, one for the approximation and second for the details of the sequence[1]. The DWT of an N point sequence x(n) is given by

$$\mathrm{DWT}\{x[n]\} = W_\phi(j_0,k) + W_\psi(j,k)$$

Where

$$W_\phi(j_0,k) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x[n]\,\phi_{j_0,k}[n]$$

$$W_\psi(j,k) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x[n]\,\psi_{j,k}[n] \qquad j \ge j_0$$

The x[n] can be recovered from DWT coefficients as

$$x[n] = \frac{1}{\sqrt{N}}\sum_k W_\phi(j_0,k)\phi_{j_0,k}[n] + \frac{1}{\sqrt{N}}\sum_{j=j_0}^{\infty}\sum_k W_\psi(j,k)\psi_{j,k}[n]$$

The DWT coefficients for x[n], (0≤ n ≤N-1) are computed for j=0,1.....,J-1 and k=0,1.....,2j-1. Here N is also a power of 2, i.e N=2j.

III. Id DWT implementation

Sub-band coding is a sequence of filtering and sub-sampling processes. At the first stage, the input sequence is filtered by two filters, a low pass and a high pass filter. The output of filter is decimated by a factor of 2, i.e only alternate (every other) output sample is retained at the filter output. As per the frequency domain, the output of the filters occupy the frequency region below and above half the sampling frequency. But due to the filter responses the two outputs may have overlapping frequency region [4].

At the start, there are two output sequences of length each N/2 corresponding to low and high pass frequencies. In the next stage, the output of the early stage low pass filter is filtered by the same low and high pass filters and decimated by a factor of 2 and give two length N/4 output sequences. The above process of filtering the output of the low pass filter and decimating can be repeated M number of times. In this octave-band sub-band coding, each time only the low pass region is split into two halves. The sub band coding where the input signal is decomposed into octave frequency bands is known as the analysis stage.

At the reconstruction or synthesis stage, the process is repeated in the reverse direction using another set of low and high pass filters. In the beginning the output of the low pass filter at the Mth stage is up sampled by 2 (inserting zero value between samples) and then filtered by a synthesis low pass filter. Similarly, the output of the high pass filter at the M stage is up-sampled by 2 and filtered by a synthesis high pass filter. Add the two outputs. The processes is repeated until a sequence of length N is obtained, which is the reconstructed sequence.

Because of filtering and sub sampling by 2, the filtered output signals suffer from aliasing distortion. Similarly, up sampling followed by filtering causes image frequency. Hence the use of low pass and high pass filters with arbitrary frequency responses will make a synthesized signal that suffers from aliasing and image distortions even without any signal quantization. The problem can be negated if the analysis and synthesis filters satisfy certain properties. Such filters bands are known as quadrature mirror filter bank[5].

The DWT of a sequence x[n] consists of the scaling and wavelet coefficients. Since the scaling functions are nested and can be express the scaling function at scale j as a linear combination of the scaling functions in scale j+1,

$$\phi[2^j n - k] = \sum_m h_0[m]\sqrt{2}\phi[2(2^j n - k) - m]$$

In above equation, let l=m+2k. Then

$$\phi[2^j n - k] = \sum_l h_0[l - 2k]\sqrt{2}\phi(2^{j+1} n - l)$$

Similarly, the wavelet function at scale j can be written in terms of wavelet functions at scale j+1,

$$\psi[2^j n - k] = \sum_l h_1[l - 2k]\sqrt{2}\psi(2^{j+1} n - l)$$

In both the above equations the factor √2 is the normalization factor so that all scaling functions and wavelets have unit energy [3].

The scaling coefficients of the DWT of the input sequence at scale j can be expressed in terms of scaling coefficients at scale j+1 as

$$W_\phi(j,k) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x[n]\phi[2^j n - k]$$

$$W_\phi(j,k) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x[n]\sum_{l} h_0[l-2k]\sqrt{2}\phi(2^{j+1}n-l)$$

$$\sum_{l} h_0[l-2k]\left\{\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x[n]\sqrt{2}\phi[2^{j+1}n-l]\right\}$$

But,

$$\frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x[n]\sqrt{2}\phi[2^{j+1}n-l] = W_\phi(j+1,l)$$

Therefore,

$$W_\phi(j,k) = \sum_{l} h_0\,[l-2k]W_\phi(j+1,l)$$

Similarly the detail or wavelet coefficients of the DWT of the input sequence at scale j in terms of scale j+1 as

$$W_\psi(j,k) = \sum_{l} h_1\,[l-2k]W_\psi(j+1,l)$$

The interpretation of above equations is the scaling coefficients at scale j is obtained by convolving or (filtering) the scaling coefficients at scale j+1 by a filter and retaining every other output sample. The filter with impulse

$$W_\psi(j,k) = \sum_{l} h_1\,[l-2k]W_\psi(j+1,l)$$

The interpretation of above equations is the scaling coefficients at scale j is obtained by convolving or (filtering) the scaling coefficients at scale j+1 by a filter $h_0(-n)$ and retaining every other output sample. The filter with impulse response $h_0(-n)$ is the time reverse version of the filter $h_0(n)$. Similarly the wavelet coefficient at scale j can be interpreted as being obtained by filtering the wavelet coefficients at scale j+1 by a filter $h_1(-n)$ and retaining every other output sample. The process of filtering followed by sub-sampling by 2 to as many stages as required. But in practice the number of stages of iteration is fixed at some value such that at least as many samples left as there are taps in the filters.

To reconstruct the sequence from its scaling and wavelet coefficients, start at the highest scale, up-sample the scaling and wavelet coefficients by 2, filter them through two filters $g_0(-n)$ and $g_1(-n)$ add the two filter outputs. This results in the reconstruction of the scaling coefficients at the next lower scale. This process is continued until the full length signal is obtained. The two reconstruction or synthesis filters are simply the time reversed versions of the corresponding analysis filters.

For perfect reconstruction synthesize the signal exactly from its DWT, the FIR filters in the filter bank must satisfy certain conditions. Earlier it was noted that the distortions that will occur are the aliasing and image distortions. To rectify this problem, the synthesis filters must possess the power complimentary or smith-Barnwell property, which is

$$|G_0(e^{j\omega})|^2 + |G_1(e^{j\omega})|^2 = 2$$

$G_0(e^{j\omega})$ and $G_1(e^{j\omega})$ are the discrete-time Fourier transform of the synthesis filters $g_0[n]$ and $g_1[n]$ respectively. The wavelets used in the DWT are either orthogonal or bi-orthogonal. Depending on the type of the DWT that is being used, the filter bank must satisfy a set of conditions.

In a two channel perfect reconstruction orthogonal DWT, the FIR filters used in the filter bank possess the following properties[6].

– The filter length is even.
– The filter $g_0[n]$ and $g_1[n]$ satisfy the power complimentary condition. Similarly, the filter pairs $\{h_0[n], h_1[n]\}$, $\{h_0[n], g_1[n]\}$ and $\{g_0[n], h_1[n]\}$ satisfy the power complimentary condition.
– The filters $g_0[n]$ and $h_0[n]$ are time-reversed versions of each other, i.e., $h_0[n] = g_0[-n]$.
– The filters $g_1[n]$ and $h_1[n]$ are time-reversed versions of each other.
– The filters $h_1[n]$ and $h_0[n]$ satisfy the condition $h_1[n] = (-1)^{n+1}h_0[L-1-n]$.
– The filters $g_1[n]$ and $g_0[n]$ are time-reversed and modulated versions of each other, i.e., $g_1[n] = (-1)^n g_0[L-1-n]$.
– Finally, $\sum_n h_0[n]=\sum_n g_0[n]=\sqrt{2}$

The filters g0[n] and h0[n] are low-pass , while g1[n] and h1[n] are high-pass. The sum of the high pass filter coefficients equals zero.

IV. Result

Simulations are done using MATLAB for the designer of FIR filters.

A. Parameters of Low Pass Filter

– Filter types= LPF
– Sampling Rate=20kHz
– $Gain_{passband}$ =1dB
– $Gain_{stopband}$ = -60dB
– Transition band=4.6kHz -5.5kHz.

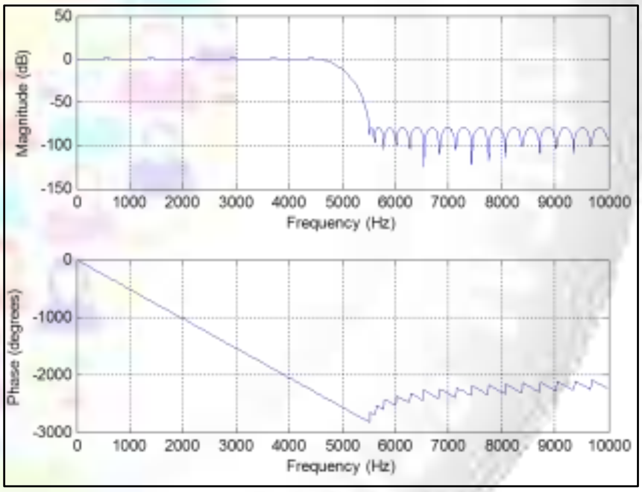

Fig. 1: Shows magnitude and phase response graph of LPF

B. *Parameters of High Pass Filter*

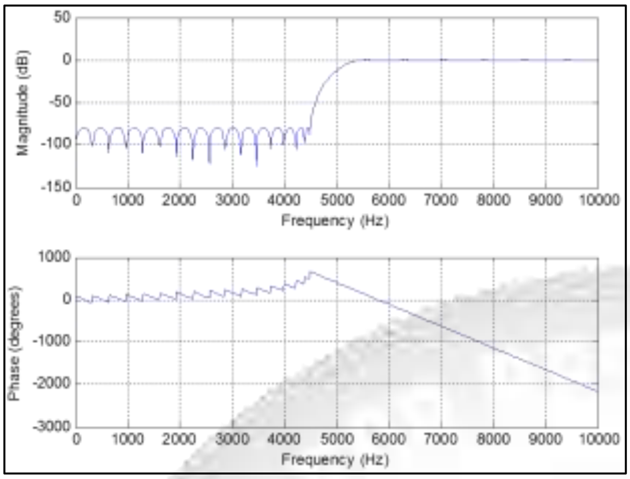

Fig. 2: Shows magnitude and phase response graph of HPF

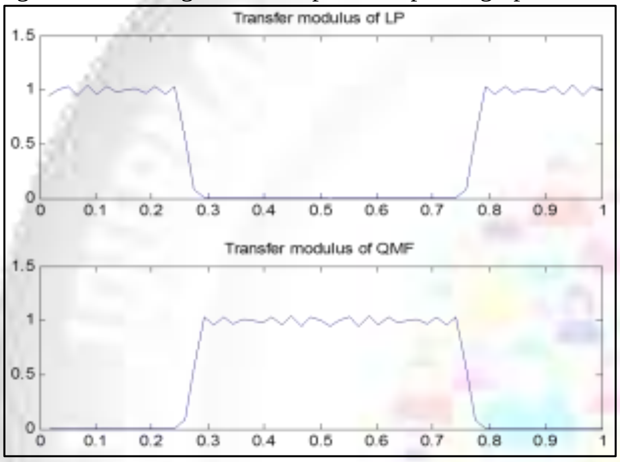

Fig. 3: Shows transfer function of QMF Low pass & High Pass filters

- Filter types= HPF
- Sample rate=66kHz
- $Gain_{passband}$ =1dB
- $Gain_{stopband}$ = -60dB
- Transition band=6.01kHz -15.05kHz.

V. CONCLUSION

Filter have advantages in different domain so, in this work the design of low pass and high pass filters completed. FIR digital filters should be choosing to use when calculations accuracy is the much requirement. In another way on the base of considering the calculations speed and calculations accuracy. The FIR low pass filter is the optimal solution has both accuracy and speed.

The design of synthesis filters follows the power complimentary property. It enable to synthesis the signal exactly. The FIR low pass filter design enables the design of analysis high pass filter along with Low and high pass synthesis filter.

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